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CONTROL OF SLOWLY VARYING LPV SYSTEMS: AN APPLICATION TO FLIGHT CONTROL *

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Abstract

Recent results in parameter-dependent control of linear parameter-varying (LPV) systems are applied to the problem of designing gain-scheduled pitch rate controllers for the F-16 VISTA (Variable-Stability In-Flight Simulator Test Aircraft). These methods, based on parameter-dependent quadratic Lyapunov functions, take advantage of known *a priori* bounds on the parameters' rates of variation (the bounds may themselves be parameter-varying). The controller achieves an induced- \mathcal{L}_2 -norm performance objective; Level 1 flying qualities are predicted. Sub-optimal solutions are obtained by solving a convex optimization problem described by linear matrix inequalities (LMIs). Incorporation of *D-K* iteration with "constant *D*-scales" provides robustness to time-varying uncertainty. Parameter-varying performance weights are used to shape the desired performance at different points in the design envelope.

1 Introduction

The area of analysis and control of *linear parameter-varying* (LPV) systems has received much recent attention, primarily in order to develop systematic techniques for gain-scheduling. These systems resemble linear systems that depend on one or more time-varying parameters; nonlinear systems are often modelled in this form via a parameterized family of linearizations. The analysis of LPV systems differ from that of linear *time-varying* (LTV) systems in that it considers whole *families* of parameter trajectories; moreover, the parameter values are available

only in real time, not in advance.

The classical approach to gain-scheduled \mathcal{H}_∞ control involves designing an (LTI) \mathcal{H}_∞ controller for each of a parameterized family of linearizations and then interpolating controller gains by operating condition. This heuristic approach yields satisfactory results if the parameters are sufficiently "slowly-varying."¹⁴ Early results in so-called "LPV synthesis" explicitly account for this time-variation using scaled small-gain arguments^{3,12} or single quadratic Lyapunov functions (SQLF);^{4,6,8} those designs tend to be conservative, though, partly because they allow the parameters to vary arbitrarily quickly. More recent results^{2,7,16,17} use parameter-dependent Lyapunov functions (PDLF) to factor in *a priori* bounds on the parameters' rates of variation, reducing this conservatism.

In this paper the PDLF technique^{7,16,17} is incorporated into the *D-K* iteration framework in order to design robust, parameter-varying pitch rate controllers for the F-16 VISTA (Variable Stability In-Flight Simulator Test Aircraft). Sub-optimal solutions are obtained by solving a convex optimization problem described by a system of linear matrix inequalities (LMIs), for which efficient algorithms are available. Parameter-varying performance weights are used to smoothly vary the desired performance within the design envelope.

The remainder of this paper is organized as follows. Section 2 reviews some results on the control of LPV systems. Section 3 provides a robustness test for LPV systems. Section 4 applies these techniques to the F-16 VISTA. Section 5 summarizes the paper and discusses future prospects.

This paper uses standard notation. In addition, $\mathcal{S}^{n \times n}$ denotes the set of real, symmetric, $n \times n$ matrices. For any matrix $X \in \mathcal{S}^{n \times n}$, $X > 0$ and $X < 0$ respectively denote positive-definiteness (all of its eigenvalues are positive) and negative-definiteness

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(all of its eigenvalues are negative). Continuously differentiable functions are called \mathcal{C}^1 , and $\mathcal{C}^1(V, W)$ denotes the set of \mathcal{C}^1 functions from V to W .

2 Control of LPV Systems

This section reviews some results on the control of continuous-time, linear parameter-varying systems. The reader may consult the applicable references^{7,16,17} (and the papers cited therein) for details.

An LPV system $G(s, \rho)$ is a finite-dimensional linear system

$$\begin{bmatrix} \dot{x}(t) \\ e(t) \end{bmatrix} = \begin{bmatrix} A(\rho(t)) & B(\rho(t)) \\ C(\rho(t)) & D(\rho(t)) \end{bmatrix} \begin{bmatrix} x(t) \\ d(t) \end{bmatrix} \quad (2.1)$$

whose state-space data are known continuous functions of time-varying parameters denoted by $\rho \in \mathbf{R}^s$. The s parameter values are not known in advance; rather, they are measured in real-time. Assume that the bounded vector-valued parameter signal $\rho \in \mathcal{L}_\infty$ is a piecewise- \mathcal{C}^1 function of time and that there exists a compact set $\mathcal{P} \subset \mathbf{R}^s$ for which $\rho(t) \in \mathcal{P}$ for all $t \geq 0$.

Also assume that the rates of variation of the first \bar{s} parameters $\rho_1, \dots, \rho_{\bar{s}}$ are each bounded in magnitude by known positive scalars $\nu_1, \dots, \nu_{\bar{s}}$, i.e.,

$$|\dot{\rho}_i(t)| \leq \nu_i \text{ for all } t > 0 \text{ (} i = 1, \dots, \bar{s} \text{)}.$$

Denote these \bar{s} rate-limited parameters and rate bounds by $\bar{\rho} := (\rho_1, \dots, \rho_{\bar{s}}) \in \mathbf{R}^{\bar{s}}$ and $\nu := (\nu_1, \dots, \nu_{\bar{s}}) \in \mathbf{R}^{\bar{s}}$, respectively. Now ν may itself be a continuous function of the parameters ρ , so that parameter trajectories must obey the differential inclusion

$$|\dot{\bar{\rho}}_i(t)| \leq \nu_i(\rho(t)) \text{ for all } t > 0 \text{ (} i = 1, \dots, \bar{s} \text{)}.$$

Parameter trajectories satisfying the above conditions for given \mathcal{P} and ν will be called *allowable*. Note that, at the cost of added notation, one can expand the results in this section to separate upper and lower bounds of the rates of variation.

2.1 Induced \mathcal{L}_2 -norm Analysis

Given a family of allowable parameter trajectories defined by a parameter set \mathcal{P} and a rate-of-variation bound ν , one can bound the induced \mathcal{L}_2 norm of an LPV system using a parameter-dependent quadratic Lyapunov function.

Lemma 2.2 Given the LPV system in (2.1) and a performance level $\gamma > 0$, suppose there exists a matrix function $W \in \mathcal{C}^1(\mathbf{R}^{\bar{s}}, \mathcal{S}^{n \times n})$ such that $W(\bar{\rho}) > 0$ and (omitting dependence on ρ and $\bar{\rho}$)

$$\begin{bmatrix} A^T W + W A + \sum_{i=1}^{\bar{s}} \beta_i \frac{\partial W}{\partial \bar{\rho}_i} & W B & C^T \\ B^T W & -\gamma I & D^T \\ C & D & -\gamma I \end{bmatrix} < 0 \quad (2.3)$$

for all $\beta_i \in [-\nu_i(\rho), \nu_i(\rho)]$ at each parameter value $\rho \in \mathcal{P}$. Then for any allowable parameter trajectory the LTV system governed by (2.1) is exponentially stable. Furthermore, there exists $\gamma_1 \in [0, \gamma)$ for which $\|e\|_2 \leq \gamma_1 \|d\|_2$ for all $d \in \mathcal{L}_2$ (if $x(0) = 0$).

The constraints $W(\bar{\rho}) > 0$ and (2.3) represent convex *linear matrix inequality* (LMI) constraints on the variables W and γ . Although these LMIs are clearly infinite dimensional, one can compute solutions using the approximate method described in the sequel.

2.2 Output-feedback synthesis

Consider an LPV plant in the standard form

$$\begin{bmatrix} \dot{x} \\ e \\ y \end{bmatrix} = \begin{bmatrix} A(\rho) & B_1(\rho) & B_2(\rho) \\ C_1(\rho) & D_{11}(\rho) & D_{12}(\rho) \\ C_2(\rho) & D_{21}(\rho) & D_{22}(\rho) \end{bmatrix} \begin{bmatrix} x \\ d \\ u \end{bmatrix} \quad (2.4)$$

where $x \in \mathbf{R}^n$, $e \in \mathbf{R}^{n_e}$, $d \in \mathbf{R}^{n_d}$, $u \in \mathbf{R}^{n_u}$, and $y \in \mathbf{R}^{n_y}$, and other quantities are dimensioned appropriately. By assuming *regularity* (i.e., D_{12} and D_{21} are full rank) and $D_{11} = 0$, (2.4) can be transformed into⁶

$$\begin{bmatrix} \dot{x} \\ e_1 \\ e_2 \\ y \end{bmatrix} = \begin{bmatrix} A(\rho) & B_{11}(\rho) & B_{12}(\rho) & B_2(\rho) \\ C_{11}(\rho) & 0 & 0 & 0 \\ C_{12}(\rho) & 0 & 0 & I \\ C_2(\rho) & 0 & I & 0 \end{bmatrix} \begin{bmatrix} x \\ d_1 \\ d_2 \\ u \end{bmatrix} \quad (2.5)$$

The *LPV γ -Performance/ ν -Variation problem* consists of finding a parameter-varying controller

$$\begin{bmatrix} \dot{x}_c \\ u \end{bmatrix} = \begin{bmatrix} A_k(\rho, \bar{\rho}) & B_k(\rho, \bar{\rho}) \\ C_k(\rho, \bar{\rho}) & D_k(\rho, \bar{\rho}) \end{bmatrix} \begin{bmatrix} x_c \\ y \end{bmatrix}$$

(possibly dependent on $\bar{\rho}$) for which the closed-loop system (omitting dependence on ρ and $\bar{\rho}$)

$$\begin{bmatrix} \dot{x} \\ \dot{x}_c \\ e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} A_{clp} & B_{clp} \\ C_{clp} & D_{clp} \end{bmatrix} \begin{bmatrix} x \\ x_c \\ d_1 \\ d_2 \end{bmatrix}$$

satisfies the conditions of Lemma 2.2 for a desired closed-loop norm $\gamma > 0$. The following theorem gives necessary and sufficient conditions for solvability.

Theorem 2.6 Given the compact set \mathcal{P} , the vector-valued function $\nu(\rho)$, the scalar $\gamma > 0$, and the open-loop system (2.5), the LPV γ -Performance/ ν -Variation problem is solvable if and only if there exist matrix functions $X \in \mathcal{C}^1(\mathbf{R}^s, \mathcal{S}^{n \times n})$ and $Y \in \mathcal{C}^1(\mathbf{R}^s, \mathcal{S}^{n \times n})$ such that (omitting dependence on ρ and $\bar{\rho}$)

$$\begin{bmatrix} \tilde{A}^T X + X \tilde{A} - \gamma C_2^T C_2 + \sum_{i=1}^s \pm \nu_i \frac{\partial X}{\partial \bar{\rho}_i} & X B_{11} & C_1^T \\ B_{11}^T X & -\gamma I & 0 \\ C_1 & 0 & -\gamma I \end{bmatrix} < 0 \quad (2.7)$$

$$\begin{bmatrix} \hat{A} Y + Y \hat{A}^T - \gamma B_2 B_2^T - \sum_{i=1}^s \pm \nu_i \frac{\partial Y}{\partial \bar{\rho}_i} & Y C_{11}^T & B_1 \\ C_{11} Y & -\gamma I & 0 \\ B_1^T & 0 & -\gamma I \end{bmatrix} < 0 \quad (2.8)$$

$$\begin{bmatrix} X & I \\ I & Y \end{bmatrix} > 0 \quad (2.9)$$

for all $\rho \in \mathcal{P}$, where

$$\begin{aligned} \tilde{A} &= A - B_{12} C_2, & C_1^T &= [C_{11}^T \ C_{12}^T] \\ \hat{A} &= A - B_2 C_{12}, & B_1 &= [B_{11} \ B_{12}] \end{aligned}$$

The notation $\pm(\cdot)_i$ indicates that the inequalities must hold for every combination of $+(\cdot)_i$ and $-(\cdot)_i$. Therefore, (2.7) and (2.8) each represent 2^s LMI's.

Controllers specified by arbitrary parameter-varying solutions $X(\bar{\rho})$ and $Y(\bar{\rho})$ typically depend explicitly on the parameter derivative $\dot{\bar{\rho}}$. On the other hand, Becker⁷ shows that in the case where the conditions of Theorem 2.6 are satisfied when X or Y is *constant* with respect to $\bar{\rho}$, one can derive formulas for strictly proper ($D_k = 0$) controllers that are independent of $\dot{\bar{\rho}}$. Specifically,

$$\begin{aligned} A_k &= N^{-1}[A^T + X(A + B_2 F + L C_2)Y + Z]M^{-T} \\ B_k &= N^{-1}X L \\ C_k &= F Y M^{-T} \end{aligned}$$

where

$$\begin{aligned} F &= -[\gamma B_2^T Y^{-1} + C_{12}] \\ L &= -[\gamma X^{-1} C_2^T + B_{12}] \\ Z &= [X(B_1 B_1^T + L B_{12}^T) + (C_1^T C_1 + C_{12}^T F)Y]/\gamma \end{aligned}$$

and M & N are chosen as follows. A constant X and parameter-varying $Y(\bar{\rho})$ admit the controller defined by choosing

$$M = I - Y X, \quad N = I$$

Similarly, a parameter-varying $X(\bar{\rho})$ and constant Y admit the controller defined by choosing

$$M = Y^{-1} - X, \quad N = Y$$

Holding *both* X and Y constant recovers the conservative SQLF controller,^{6,8} which allows for arbitrarily fast parameter variation.

2.3 Computing Solutions

The infinite-dimensionality of the inequalities in Theorem (2.6) demands approximate methods of solution for the sake of practical computation. One such method follows:^{16,17}

Pick scalar basis functions $\{f_i \in \mathcal{C}^1(\mathbf{R}^s, \mathbf{R})\}_{i=1}^{N_X}$ and $\{g_j \in \mathcal{C}^1(\mathbf{R}^s, \mathbf{R})\}_{j=1}^{N_Y}$, and search over those $X(\bar{\rho})$'s and $Y(\bar{\rho})$'s that are linear combinations

$$X(\bar{\rho}) = \sum_{i=1}^{N_X} f_i(\bar{\rho}) X_i, \quad Y(\bar{\rho}) = \sum_{j=1}^{N_Y} g_j(\bar{\rho}) Y_j$$

using constant matrices $\{X_i \in \mathcal{S}^{n \times n}\}_{i=1}^{N_X}$ and $\{Y_j \in \mathcal{S}^{n \times n}\}_{j=1}^{N_Y}$. Then (2.7)-(2.9) can be rewritten as (omitting dependence in ρ and $\bar{\rho}$)

$$\begin{bmatrix} \left\{ \begin{aligned} &\sum_{i=1}^{N_X} (\tilde{A}^T X_i + X_i \tilde{A}) f_j - \gamma C_2^T C_2 \\ &+ \sum_{k=1}^s \pm \nu_k \sum_{i=1}^{N_X} \frac{\partial f_i}{\partial \bar{\rho}_k} X_i \end{aligned} \right\} & (*) & (*) \\ & & -\gamma I & (*) \\ & & 0 & -\gamma I \end{bmatrix} < 0 \quad (2.10)$$

$$\begin{bmatrix} \left\{ \begin{aligned} &\sum_{j=1}^{N_Y} (\hat{A} Y_j + Y_j \hat{A}^T) g_j - \gamma B_2 B_2^T \\ &- \sum_{k=1}^s \pm \nu_k \sum_{j=1}^{N_Y} \frac{\partial g_j}{\partial \bar{\rho}_k} Y_j \end{aligned} \right\} & (*) & (*) \\ & & -\gamma I & (*) \\ & & 0 & -\gamma I \end{bmatrix} < 0 \quad (2.11)$$

$$\begin{bmatrix} \sum_{i=1}^{N_X} f_i X_i & I \\ I & \sum_{j=1}^{N_Y} g_j Y_j \end{bmatrix} > 0 \quad (2.12)$$

where the shorthand $(*)$ denotes the implied transposes. The problem thus consists of finding real, symmetric matrices $\{X_i\}_{i=1}^{N_X}$ and $\{Y_j\}_{j=1}^{N_Y}$ that satisfy the above inequalities for all $\rho \in \mathcal{P}$, still an infinite-dimensional problem.

Now approximate the parameter set \mathcal{P} by a grid of L points $\{\rho_k \in \mathbf{R}^s\}_{k=1}^L$, defining $\{\bar{\rho}_k \in \mathbf{R}^s\}_{k=1}^L$ accordingly, and solve the inequalities (2.10)-(2.12) at these grid points. The conditions represent up to $L(2^{s+1} + 1)$ LMIs in the $N_X + N_Y$ matrix variables $\{\{X_i\}_{i=1}^{N_X}, \{Y_j\}_{j=1}^{N_Y}\}$. Note that minimizing γ is a convex optimization problem, since the inequalities are affine in γ as well.

3 Robust LPV Systems

This section derives a simple robustness test and proposes an *ad hoc* method for making the preceding control synthesis robust. Consider the analysis of an LPV system

$$\begin{bmatrix} z \\ e \end{bmatrix} = G(s, \rho) \begin{bmatrix} w \\ d \end{bmatrix}$$

put in feedback with a block-diagonal, memoryless linear time-varying (uncertainty) operator

$$w(t) = \Delta(t)z(t)$$

where $w, z \in \mathbf{R}^{n_z}$, $d \in \mathbf{R}^{n_d}$, and $e \in \mathbf{R}^{n_e}$. Assume that the block structure of Δ is consistent with a set $\Delta \subset \mathcal{S}^{n_z \times n_z}$. Define a corresponding set of scalings

$$\mathbf{S}_\Delta := \{S \in \mathcal{S}^{n_z \times n_z} : \Delta S = S \Delta \text{ for all } \Delta \in \Delta\}$$

An elementary small-gain argument establishes the following sufficient conditions for robustness to (arbitrarily quickly) time-varying uncertainty.

Proposition 3.1 Suppose there exists a continuous matrix function $S : \mathbf{R}^s \rightarrow \mathbf{S}_\Delta$ such that for any allowable parameter trajectory $\rho : \mathbf{R} \rightarrow \mathbf{R}^s$ the resulting LTV system

$$G_S(s, \rho) = \begin{bmatrix} S(\rho) & 0 \\ 0 & I_{n_e} \end{bmatrix} G(s, \rho) \begin{bmatrix} S^{-1}(\rho) & 0 \\ 0 & I_{n_d} \end{bmatrix}$$

is exponentially stable and $\|G_S(s, \rho)\|_{i2} \leq \gamma$ (given zero initial conditions). Then for any allowable parameter trajectory ρ and any Δ satisfying

$$\bar{\sigma}(\Delta(t)) \leq 1/\gamma$$

the LTV system $F_u(G(s, \rho), \Delta)$ also is exponentially stable and $\|F_u(G(s, \rho), \Delta)\|_{i2} \leq \gamma$ for zero initial conditions.

This suggests that one can design *robust* LPV controllers by alternating (in obvious fashion) between controller synthesis and computation of parameter-varying scaling matrices $S(\rho)$, as in *D-K* iteration. The design example in this paper uses an *ad hoc* choice of $S(\rho)$: “zeroth-order” fits to frequency-dependent *D*-scales that are obtained at each “frozen” grid point by applying the appropriate μ -Tools commands for closed-loop analysis. There is also a more rigorous iterative approach² that will not be addressed in this paper.

4 A Design Example

This section presents an application of LPV synthesis to the pitch rate control of the F-16 VISTA over a specified flight envelope; a similar problem has been addressed¹⁵ using a small-gain method. The physical plant and certain design weights are parameter-varying, and a *D-K* iteration-like process is employed to enhance robust performance.

4.1 Plant Modelling

This design uses the standard short-period equations of motion⁹

$$\begin{bmatrix} \dot{\alpha} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} Z_\alpha(\rho) & 1 \\ M_\alpha(\rho) & M_q(\rho) \end{bmatrix} \begin{bmatrix} \alpha \\ q \end{bmatrix} + \begin{bmatrix} Z_{\delta_e}(\rho) \\ M_{\delta_e}(\rho) \end{bmatrix} \delta_e \quad (4.1)$$

$$\rho = \bar{\rho} = (M, h)$$

ignoring the aerodynamic effects of the trailing edge flaps. Only the longitudinal dynamics of the aircraft are considered; the roll, yaw, and sideslip angles are assumed to be zero. In (4.1) the states (α, q) , input δ_e , and parameters (M, h) respectively denote angle-of-attack & pitch rate, elevator deflection, and Mach number & altitude.

The Flight Dynamics Directorate of the Wright Laboratory uses a high-fidelity, six degree-of-freedom, nonlinear model to simulate the F-16 VISTA.¹ This simulation model includes accurate descriptions of the propulsion system, actuators, sensors, disturbances, payload, atmosphere, rigid-body equations of motion, and aerodynamics for a wide range of Mach numbers, altitudes, and angles of attack.

The LPV short-period model’s state space data (the dimensional coefficients Z_α , M_α , M_q , Z_{δ_e} , and M_{δ_e}) were obtained by trimming and linearizing the nonlinear model at level flight for the flight conditions marked in Table 1. The design region \mathcal{P} was chosen accordingly, and these 21 grid points were used for the controller synthesis.

$h \backslash M$	0.35	0.45	0.55	0.65	0.75	0.85
25000 ft			X	X	X	X
15000 ft		X	X	X	X	X
5000 ft	X	X	X	X	X	X
1000 ft	X	X	X	X	X	X

Table 1: Grid points used for modeling & synthesis

Data on the excess thrust and rate-of-climb of the

F-16 VISTA suggest the bounds

$$\begin{aligned}\nu_1(\rho) &= 2 \left(\frac{|\dot{V}_T|_{max}}{a(h)} + M^2 \left| \frac{da(h)}{dh} \right| \right) \geq |\dot{M}(t)| \\ \nu_2(\rho) &= a(h)M \geq |\dot{h}(t)|\end{aligned}$$

on the parameters' rates of variation, where $a(h)$ denotes the speed of sound as a function of altitude and $V_T = a(h)M$ denotes the aircraft's true velocity. Note that these bounds are conservative; achieving them would require vertical flight, for example.

4.2 Problem Setup

The objective here is to design for the F-16 VISTA a pitch-rate controller that provides robust command tracking with predicted Level 1 handling qualities.¹¹ Time-domain specifications for pitch-rate response are illustrated in Fig. 1 and listed in Table 2. Note that the "rise-time" parameter Δt varies with the true velocity V_T (in ft/sec).

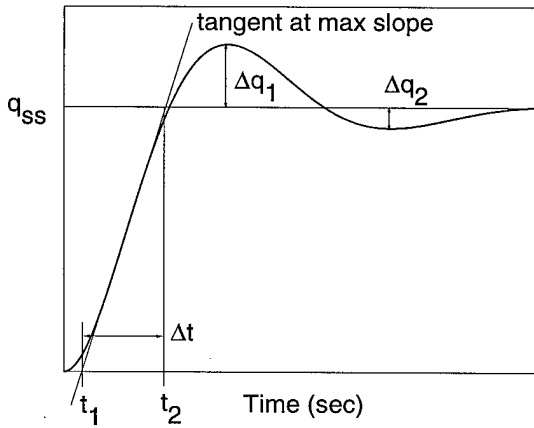


Figure 1: Pitch rate handling qualities specifications

Parameter	Level 1	Level 2
max t_1 (sec)	0.12	0.17
max $\Delta q_2 / \Delta q_1$	0.30	0.60
max Δt (sec)	$500/V_T$	$1600/V_T$
min Δt (sec)	$9.0/V_T$	$3.2/V_T$

Table 2: Pitch rate handling qualities specifications

This design uses the model-matching control structure shown in Fig. 2. The second-order reference model

$$G_{ref}(s) = \frac{\omega_n^2(Ts + 1)}{s^2 + 2\zeta\omega_n s + \omega_n^2},$$

$$\omega = 4 \text{ rad/s}, \zeta = 0.6, 1/T = 10 \text{ rad/s}$$

meets Level 1 flying qualities over the entire design envelope. The first-order, parameter-varying command and performance weights

$$\begin{aligned}W_r(s, \rho) &= \frac{s + 100}{100} \frac{10}{s + 10} q_{max}(\rho) \\ W_p(s, \rho) &= \frac{s + 80}{80} \frac{4}{s + 4} \frac{1}{0.05 q_{max}(\rho)}\end{aligned}$$

reflect a uniform 10 rad/s command bandwidth, a maximum pitch-rate command q_{max} that varies across the design envelope as shown in Table 3, and steady-state tracking within 5%.

$h \setminus M$	0.35	0.45	0.55	0.65	0.75	0.85
25000 ft			13	17	20	22
15000 ft		12	15	18	21	23
5000 ft	10	13	16	19	22	24
1000 ft	11	14	17	20	23	25

Table 3: Max. pitch-rate command q_{max} (deg/s)

The design plant also includes actuator dynamics, additive sensor noise, multiplicative input uncertainty, and penalties on the elevator deflection angle and rate. The first-order actuator model

$$G_a(s) = \frac{20.2}{s + 20.2}$$

reflects a bandwidth of 20.2 rad/s. The control weight

$$W_\delta = \begin{bmatrix} 21 & 0 \\ 0 & 70 \end{bmatrix}$$

reflects elevator deflection angle and rate limits of 21 deg and 70 deg/s, respectively. The parameter-dependent noise weight

$$W_n(\rho) = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.03 q_{max}(\rho) \end{bmatrix}$$

anticipates 0.5 deg of measurement error for α and 0.03 q_{max} deg/s (about 3%) for q . The uncertainty weight

$$W_u = 0.1$$

represents 10% parametric and/or dynamic modeling error.

4.3 Design Results

Several D - K iterations (using LPV synthesis) were performed in MATLAB on the 7th-order design plant. LMILab¹⁰ was used to solve the controller

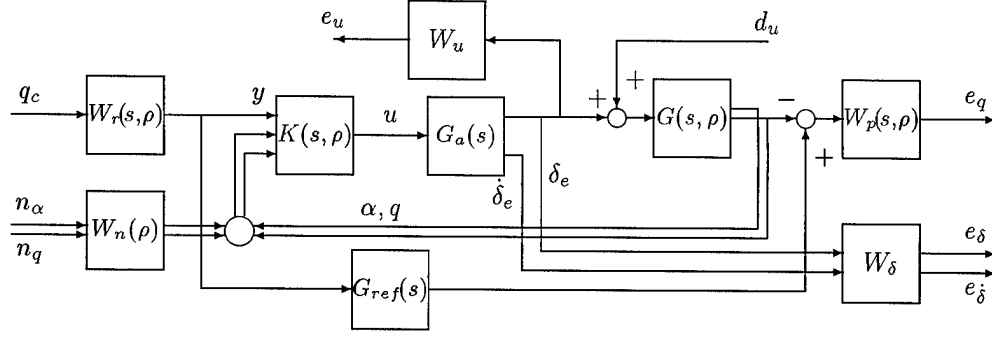


Figure 2: The design plant

synthesis LMIs, and μ -Tools⁵ was used for closed-loop robustness analysis. The elementary basis functions

$$[f_1(\bar{\rho}) \ f_2(\bar{\rho}) \ f_3(\bar{\rho})] = [g_1(\bar{\rho}) \ g_2(\bar{\rho}) \ g_3(\bar{\rho})] = [1 \ M \ h]$$

selected according to previous experience,^{2,13,16,17} were used to vary the matrices $X(\bar{\rho})$ and $Y(\bar{\rho})$ with various degrees of complexity: constant X & Y , varying $X(\bar{\rho})$ & constant Y , constant X & varying $Y(\bar{\rho})$, and varying $X(\bar{\rho})$ & $Y(\bar{\rho})$.

D-K Iter. #	X, Y			$X, Y(\bar{\rho})$		
	γ	$\bar{\gamma}_\infty$	$\bar{\mu}$	γ	$\bar{\gamma}_\infty$	$\bar{\mu}$
1	2.60	1.90	1.79	1.49	1.14	1.13
2	2.43	1.75	1.73	1.22	1.05	1.04
3	2.42	1.74	1.72	1.19	1.04	1.03
4	2.42	1.74	1.72	1.18	1.04	1.03

Table 4: Closed-loop performance levels

The closed-loop time-varying performance level γ achieved using Theorem 2.6 is shown in Table 4; constraining the rate of variation of M and h clearly offers a significant improvement in performance. Also included are the maximum “frozen-point” (i.e. gain-scheduled) \mathcal{H}_∞ norms

$$\bar{\gamma}_\infty = \max_{\rho \in \mathcal{P}} \|G_{clp}(s, \rho)\|_\infty$$

and structured singular values (cf. Figure 3)

$$\bar{\mu} = \max_{\rho \in \mathcal{P}} \sup_{\omega \in \mathbf{R}} \mu[G_{clp}(j\omega, \rho)]$$

obtained via pointwise \mathcal{H}_∞ and μ -analysis of the closed-loop systems. These indicate the controller’s robustness at constant flight conditions. The two designs with varying $X(\bar{\rho})$ offer no appreciable improvement over the corresponding constant- X designs, so they have been omitted.

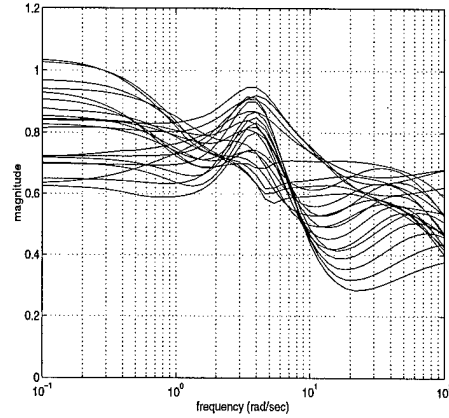


Figure 3: Closed-loop $\mu(j\omega)$ after 4th iteration

4.4 Nonlinear Simulation

Figure 4 shows the results of a high-fidelity, parameter-varying, nonlinear simulation of the closed-loop step response, which demonstrates predicted Level 1 flying qualities. The aircraft is initially perturbed from trimmed, level flight at $M = 0.75$ and $h = 5000$ ft. The response of the constant- X, Y controller is also shown, for comparison.

5 Conclusions

Recent results in parameter-dependent control of linear parameter-varying systems are applied to the problem of robust, gain-scheduled pitch rate control for the F-16 VISTA. Using parameter-dependent Lyapunov functions, *a priori* bounds on the parameters’ rates of variation, LMI-based convex optimization, and parameter-varying design weights, this method achieves an induced- \mathcal{L}_2 -norm performance objective while predicting Level 1 flying qual-

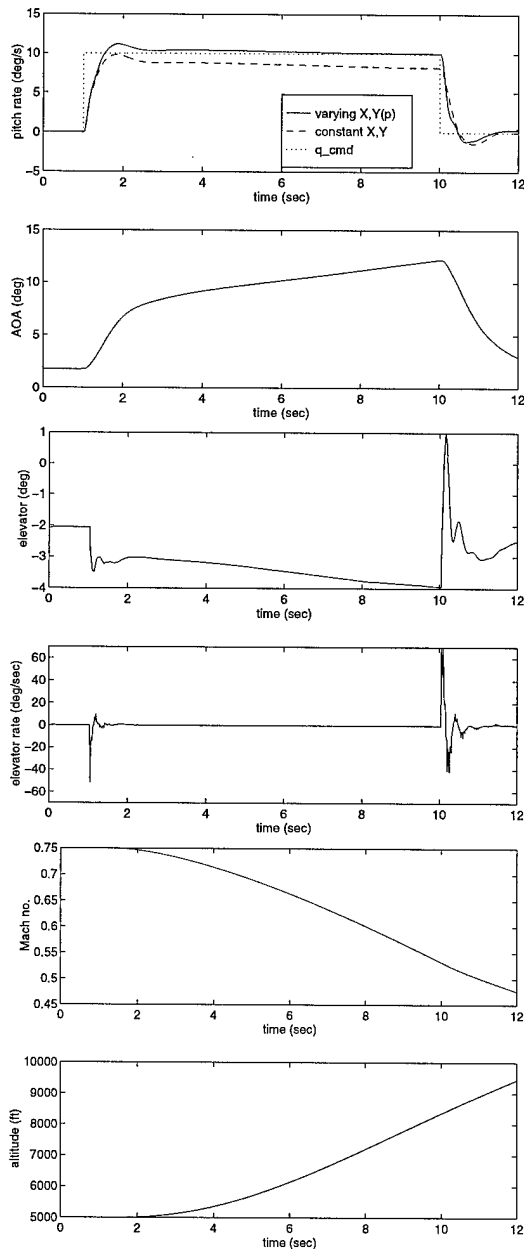


Figure 4: Parameter-varying nonlinear simulation

ities throughout the design envelope. Straightforward D - K iteration with "constant D -scales" provides robustness to time-varying uncertainty. Ongoing research includes expanding the parameter set (to encompass the full flight envelope) and including a parameter-varying reference model (while maintaining Level 1 flying qualities), and incorporating the robustness scales into the synthesis LMIs.

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